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set and to deduct a fixed percentage for failure to do this. They should insist that the candidate should show on his paper that he understands how to check his work. Of course this will not be done without adequate notice to the schools.

When it is done we shall have text-books adequate for such a requirement, teachers will systematically teach checks, and the result will be a greatly increased accuracy and efficiency in our mathematical work.

STUYVESANT HIGH SCHOOL,
NEW YORK, N. Y.

THE AIMS IN TEACHING GEOMETRY AND HOW TO ATTAIN THEM.*

BY W. E. BOND.

The aims in teaching geometry should be, according to my views:

1. That the pupil should acquire an accurate, thorough knowledge of geometrical truths.
2. That he should develop the power of original, logical, geometrical reasoning.
3. That he should acquire a habit of thought which will give him a practical sagacity; which will develop his judgment, increase his resourcefulness, and fit him to cope more successfully with the many and varied problems of his after life; which will teach him to take a many-sided view of things, so that if the avenue of attack is blocked, he shall be able to promptly, cheerfully and successfully attack from another quarter.

In my opinion the last-mentioned aim is by far the most important; the first, as an end in itself, I deem the least important of the three.

Having stated the aims to be attained, I now turn to a discussion of the difficulties which, in my own experience, have presented themselves, and have tended to hinder the attainment of these aims.

In my early experience as a teacher of geometry the first difficulty I met was the seeming lost or dazed condition of so many of my pupils for the first few weeks of the term. They did not see what we were trying to do. Their previous work in mathematics had been so different from this that it hindered rather than helped them. Not understanding what they were trying to accomplish, they naturally did not make much headway in accomplishing it. With some the mists cleared away in a few days; with others in a few weeks; while still others remained in the densest kind of a fog for the whole term, or dropped out, discouraged, by the way.

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* Read at the meeting of the Syracuse Section, December 29, 1908.

The second difficulty I encountered was the *memorizing* of demonstrations. In spite of changing the lettering of the figures, changing the shape of the figures, turning them upside down, etc., a large number, especially among the girls, would depend more upon *memory* than upon *reason*. Even where the demonstration as a whole seemed to be logically worked out, a little questioning would often bring out the fact that certain positions were accepted upon the authority of the author, and had not been clearly thought out by the student.

A third difficulty was in teaching the pupil how to attack new propositions. No set rules or methods or suggestions seemed to help them much. What they needed was a *condition of mind,* which could not be taught, directly.

These were the main difficulties which at first hindered the realization of my ideals, and they have been ever since, and are today, the problems which I am trying to solve. I shall now proceed to indicate along what lines I am trying to work out these problems.

In the first place I have become convinced that the first difficulty I mentioned, the dazed condition of the pupils at first, is due to a lack of understanding of what *proof* or *demonstration* consists; of what it really means; and of the necessary process to be gone through in proving a proposition. So, first of all, I believe we should get our classes to see that proof consists in showing that the proposition in question is a special or particular case of some general truth which must have been already admitted without proof. Whether it takes one day, or three days, or a week, the teacher of geometry should not begin the regular work of the subject until every pupil is perfectly clear on this point. So that when we ask them if they will admit, without proof, certain statements which we call "axioms," every member of the class will know just *why* we ask them to admit, without proof, these truths, and just what relation these axioms are to bear to the work which is to follow.

As to the third difficulty, that of teaching pupils how to attack new propositons, I have come to the conclusion that they can learn to do only by *doing*. The first step in learning to swim is to get into the water. The teacher of Latin does not put a translation of Cæsar into the hands of his pupils; why should we start off by putting a pony into the hands of ours? We have all noticed that actual grappling with originals seems to give the greatest increase in power to cope with originals. Then why not let the work be original reasoning from the start? If in this way we can overcome the third difficulty I mentioned, the second, the memorizing tendency, will take care of itself.

So, several years ago I began experimenting along this line. I rearranged the theorems of the first book of plane geometry so that, while in a logical order, they were somewhat different from that of any text-book. I cut out most of the corollaries, the few really important ones being incorporated as theorems. I tried to arrange the theorems so that the proofs on which a theorem depends should precede it as closely as possible. Also I tried as far as practicable to grade the theorems from easy to difficult. I did not attempt to group the theorems according to any classification of the figures; for example those which prove triangles equal are numbers 4, 5, 9, 19, 31 and 32 in my list. My first experiment was with a class of about forty. The result, compared with the results of my previous teaching, was such that I have since continued it.

Each pupil is supplied at the start with a note-book, but at first with no text-book. After explaining to the class the nature of the subject-matter in geometry, or of what geometry treats, the teacher states to the class that he wishes to prove to them certain propositions about lines, angles, etc., and asks them to tell him what method of procedure he must follow. If the preliminary work has been properly done, the class will assure him that he must first get them to admit, without proof, some simple, self-evident, general truths,, and then must show that the propositions he wishes to prove are special, particular cases of these general truths which they have already admitted. Cautioning them to consider carefully and not admit anything unless it is so simple and self-evident that they cannot help admitting it, the teacher proposes, one at a time, the list of axioms which he has prepared. As fast as admitted the pupils are told to write them in their note-books; labeling them axiom 1, axiom 2, etc.

The pupils now know why these axioms have been proposed, and are anticipating the theorems which are to follow. They do not feel lost, or uncertain as to what it all means, but are ready and waiting for the course of reasoning that is to come.

The teacher now proposes theorem I, and proceeds to demonstrate or prove it to them. When all have signified their acceptance of the proof, the teacher dictates the demonstration, the pupils write it in their note-books, and are required to be able to reproduce it next day, *verbatim*. In this way the first five or six theorems are dictated by the teacher, written down and reproduced by the class. The object of this dictation and verbatim reproduction is to furnish the pupil a model so that in his future work he may follow a logical, well arranged form. The next four or five theorems are demonstrated by the teacher, but not dictated. By this time the class should be ready to begin to walk alone, the teacher gradually removing the leading strings, being careful not to discourage them by requiring more than they can do at first. In the more difficult propositions the teacher may for a time suggest the construction lines, but this prop should also be removed as soon as possible. For the first eight or ten weeks most of the advance lesson is prepared in class under the eye of the teacher. Fifteen or twenty minutes before the close of the period the advance theorem is assigned, the pupils take pencil and paper and attack the proposition, always first of all writing down what is *given* and what they are to *prove*. Let us suppose the theorem to be "Two straight lines which are parallel to the same straight line are parallel to each other," which is no. 16 in my list. The teacher passes up and down the aisles and observes the progress each is making. Those who are evidently getting a start he leaves with perhaps a word of encouragement, and halts at a desk whose occupant has as yet not written a word or drawn a construction line and asks what is the trouble. The pupil replies that he can't "get started." The teacher asks what ways he has had of proving lines parallel. The pupil names Definition of parallel lines and Theorems 7, 10, 12 and 14. The teacher observes that from such a number of ways of proving lines parallel he certainly ought to find one that can be made to apply to this case, and suggests that he try them one after the other until one is found that will work. The teacher stops again at the desk of a pupil who is started along a wrong line and asks a question which calls the pupil's attention to his error. In this way the teacher goes around the room, giving encouragement here, a hint there, but no help except where imperatively needed. At the close of the period the papers are collected, errors marked in blue pencil and returned to the pupils next day. (For this work I use a blank form somewhat similar to the *Geopad*. The average pupil will in from three to five minutes fill in one of these blanks a review demonstration which would require fifteen minutes to write out in full.)

At the beginning of the next recitation the teacher puts the figure for this theorem on the board, the different ways of demonstrating it are discussed, criticisms are invited, and questions answered. In this way the pupils form habits of study which will fit them to later prepare their lessons at home.

Upon the board, and also copied in each pupil's note-book, there should be a blank form, or table of about a dozen columns, with the following headings: Angles Equal, Angles Unequal, Lines Equal, Lines Unequal, Lines Parallel, Lines Perpendicular, Triangles Equal, etc. Each heading has a double column, the first marked "Direct Proofs," the second "Indirect Proofs." As each theorem is demonstrated it is recorded in the appropriate column; also definitions and axioms which might be used in proving the truth of the headings. The pupils soon learn to consult this list for the available proofs to be used in demonstrating new propositions, and as all the previous proofs are recorded in their appropriate columns, the pupil is almost sure if he perseveres, to find one that will apply.

By the time the first book has been finished in this way, the pupils have acquired good habits of thought and have learned how to study. They may now, if preferred, be supplied with text-books, or they may continue as in Book I. The pupils themselves, if given a voice in the matter, will invariably choose to go on without a text-book. If a text is used the demonstrations must be slightly modified to fit Book I as the pupils have had it. This they easily do, and it serves as a further check to memorizing demonstrations.

What are the disadvantages in this method of teaching geometry without a text-book? I have found only two. First, unless the teacher is watchful, pupils may fall into the habit of using mathematical terms in a loose and inaccurate way; second, pupils who have never used a text-book have more difficulty in following a text-book demonstration than those who have used one. For both these reasons I think it is advisable for the *inexperienced* teacher to use a text for the last part of the course.

Teachers who do not feel competent or do not have the time to rearrange the theorems of Book I may take them from some other text-book to which the students do not have access. All texts cover practically the same ground in the first book.

The matter of enthusiasm and interest I have not yet mentioned, yet they are by no means the least benefits to be derived by this method. Pupils will become as much absorbed in succeeding with a demonstration as in winning a game. Time and again I have heard a student say at the successful ending of a struggle with a theorem that he had "beaten the old cat." There is always a wholesome rivalry to see who will first succeed in getting a demonstration. And the one who has to confess to a failure always feels it a keen disgrace.

This method encourages and develops the individuality of the pupil. It makes him an independent thinker. It gives him confidence in his own powers.

The teacher, also, is more enthusiastic, and becomes more interested in his work. He becomes better acquainted with his pupils and their individual peculiarities of mind. He learns what their difficulties really are, and hence can aid them more intelligently.

I have tried this method on more than twenty classes, ranging in numbers from twenty to sixty. I have also during this time started several different assistants in this method, usually inexperienced teachers fresh from college. From this experience I am satisfied that any teacher who can get fair results with a text-book, can get much better results with original work.

To summarize:

Aims.

First. Acquirement of knowledge of geometrical truths.
Second. Power of logical geometrical reasoning.
Third. Develoment of sagacity.

Difficulties.

First. Dazed condition in early part of the work.
Second. *Memorizing* instead of *reasoning.*
Third. Inability to attack new propositions.

Remedies.

For the first difficulty. Preliminary discussion of the nature of proof.
For the second and third. Original work.

POTSDAM NORMAL SCHOOL,
POTSDAM, N. Y.

WHAT SHOULD BE THE AIMS IN TEACHING ALGEBRA AND HOW ATTAIN THEM.*

BY ARTHUR M. CURTIS.

The aims in teaching algebra are largely those that govern the teaching of any subject:

The first and most essential aim must be to enlist the interest of the pupil in the subject to the end that he will put forth *persistent* effort in its mastery.

From the nature of the subject there are three special aims that may well claim the attention of the teacher:

1. To increase the pupil's knowledge of mathematical shorthand.

2. To increase the pupil's knowledge of truth, mathematical truth, in preparing him for future work in college and technical schools.

3. To establish in the mind of the pupil the firm belief that he can *of himself* distinguish truth from error and to see to it that he *forms the habit of proving all things.*

The means are the following:

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* Abstract of a paper read at the meeting of the Syracuse Section, December 29, 1908.